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april/12/2006

Using Kalman Filter to observe the internal state

We have mentioned before that we can use Kalman Filter to “guess“ the internal state of a system. Just because you can see the output of a system, it doesn't mean that you know what is happening inside the system. The kalman filter allows you to take the output of the system and tell you if what's inside is blowing up or calm down.

In this section, we are going to take an example system and add a bias. If you look at the function below, notice that the output  $y$  is the combination of  $x$  and a constant value  $y_b$ . So the kalman filter will look at  $y$  and tell you what  $y_b$  as well as  $x_1$  and  $x_2$  are.

I want to also mention that since we have made up this system, we could do what ever we want with  $y_b$ . As a matter of fact, I am going to set it as 5. But keep in mind that the Kalman filter we are about to design will have no clue what  $y_b$  is, but will still guess it.

So here's the system.

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} -\mu & \omega \\ -\omega & -\mu \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} - (x_1^2 + x_2^2) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} w + \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} u$$
$$y = (1 \ 0)x + v + y_b$$

Where

$w$  and  $v$  are white noise

$y_b$  is an unknown constant bias

$u = 0$

$$x_0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

We are going to observe the zero input response.

The first thing we want to do is to linearize it.

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} -\mu & \omega \\ -\omega & -\mu \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} w + \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} u$$

Since  $y_b$  is also something we want to track, we need to make it into a state.

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{y}_b \end{pmatrix} = \begin{pmatrix} -\mu & \omega & 0 \\ -\omega & -\mu & 0 \\ 0 & 0 & e^{-4} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ y_b \end{pmatrix} + \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 0 & 0 & e^{-4} \end{pmatrix} \begin{pmatrix} u \\ w \end{pmatrix}$$

$$y = (1 \ 0 \ 1) \begin{pmatrix} x_1 \\ x_2 \\ y_b \end{pmatrix} + (e^{-4} \ e^{-4} \ e^{-4}) \begin{pmatrix} u \\ w \end{pmatrix} + v$$

You might have noticed that we didn't make  $y_b = 0$ . This is for the kalman filter sake, we must have matrices with full rank. So instead of having zero, we just use a very small number. By changing this number, it actually determines how quickly we can track the bias.

The next thing we need to do is to find the kalman gain L, we can go through the math by hand, but luckily matlab does it for us with these commands.

```
A = [-u w 0; -w -u 0; 0 0 0];
B = [1 1.0000 1.0000 1 -1.0000 1.0000 0 0 2*exp(-1)];
C = [1.0000 0 1];
D =[exp(-1) exp(-1) 2*exp(-1)];

sys = ss(A,B,C,D);
[kesk, L, P] = kalman(sys,1,1);
```

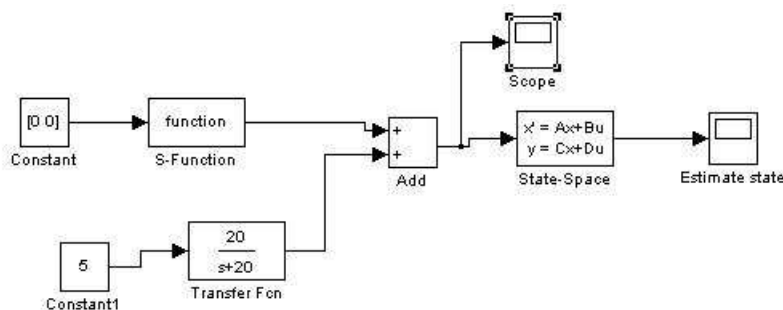
The first command ss, defines a system for us, while kalman help us find the kalman gain. Out of the three outputs kesk, L, and P, we only need the kalman gain L.

Once we have the kalman gain, we can find the output using a state space block with these state space equations.

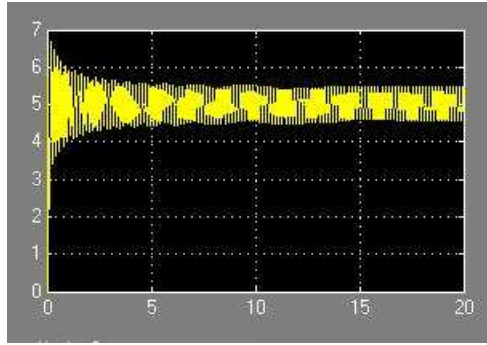
$$\dot{x}' = (A - LC)x' + Ly$$

$$x' = x'$$

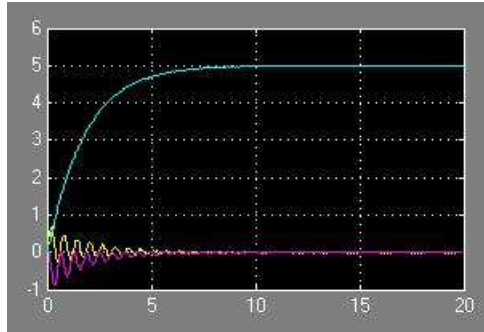
Here is a picture of what the simulink block should look like.



Take a look at the blocks. We have already covered how S-functions work from previous examples I have written. Notice how we have added a bias of 5 at the output. So here's what the Kalman filter sees. ( Taken from scope )



Now, here's what the kalman filter thinks the state as well as our bias should be.



Take a while guess which line is tracking the bias 5, and which are tracking the other states. Now, remember how we introduced the fake noise before for  $y_b$  instead of using zero. By controlling that number, it allows us to track it faster or slower. The smaller the number the slower it tracks. If I make that number something like  $e^{-3}$ , it would take much longer to converge to 5 than the picture above.

